

KISII UNIVERSITY  
BSc. ACTUARIAL SCIENCE  
LIFE CONTIGENCIES I

BACS 321  
EXAM

APRIL, 2023

ATTEMPT ALL QUESTIONS IN SECTION ONE AND ANY TWO  
OTHER QUESTIONS IN SECTION TWO.

**TIME: 2 HOURS**

**SECTION ONE (30 marks)**

## 1 Question One

a) Calculate the probability that a life now aged 30 exact will die between the ages of 55 *and* 65 both exact.

Basis: Mortality *ELT15 (Males)* (4 marks)

b) A life aged 60 exact purchases a single-premium temporary annuity. The annuity pays 8000 annually in arrears for a term of 8 *years*, ceasing on death, if earlier.

Calculate the reserve for the annuity at the end of the first policy year, using the net retrospective method and the following basis:

<i>Mortality</i>	<i>AM92 Select</i>
<i>Interest</i>	<i>4% per annum</i>
<i>Expenses</i>	<i>Ignore</i>

(4 marks)

c) A term assurance policy is issued on a life aged  $x$  for a term of 20 *years*. Under this policy a sum assured, payable immediately on death, is 10,000 for the first 10 *years* increasing to 20,000 for the subsequent 10 *years*.

i) Calculate the expected present value of the benefits. (3 *marks*)

ii) Determine the variance of the present value of the benefits. (5 *marks*)

Basis:

$$\begin{array}{ll} \text{Mortality} & \mu_x = 0.03 \text{ for all } x \\ \text{Force of Interest} & \delta = 5\% \text{ throughout} \end{array}$$

(8 *marks*)

d) A unit-linked policy has the following profit vector:

<i>Year</i>	<i>In – force profit</i>
1	–25
2	–12
3	–6
4	25
5	35

i) Calculate the reserves required in order to zeroise the losses occurring at the end of year 2 *and* 3. Assume a rate of accumulation of 8% *pa* effective, and that  $q_x = 0.01$  at each age. (2 *marks*)

ii) If the risk discount rate used is 10% *pa* effective, determine the net present value of the profits before and after zeroisation and state with reasons which of these figures you would expect to be greater. (5 *marks*)

e) Under a 10 – *year* "double endowment" assurance policy issued a group of lives aged 50, a sum assured of £10,000 is payable at the end of year of death and £20,000 paid if the life survives to the maturity date. Premium are payable annually in advance.

You are given the following the following:

<i>reserve at the start of the 8th year (per policy in force)</i>	£12,951
<i>number of policies in force at the start of the 8th year</i>	200
<i>number of deaths during the 8th year</i>	3
<i>annual premium (per policy)</i>	£1,591

i) Assuming that reserves are calculated according to the basis specified below, calculate the profit or loss arising from mortality in the 8th year. (5 marks)

ii) Comment on your results

Basis:

*Mortality* *ELT (Males)*  
*Interest* *4%pa effective*  
*Expenses* *None*

(7 marks)

## SECTION TWO (40 marks)

### 2 Question Two

a) An insurer issues a combined term assurance and annuity contract to a life aged 35. Level premiums are payable monthly in advance for a maximum of 30 years.

On death before age 65 a benefit is paid immediately. The benefit is £200,000 on death in the first year of the contract, £195,000 on death in the second year, £190,000 on death in the third year, etc, with the benefit decreasing by £5,000 each year until age 65. No benefit is payable on death after age 65.

On attaining age 65 the life receives an annuity of £10,000 *pa* payable monthly in arrears.

Calculate the monthly premium on the basis of:

*Mortality* : up to age 65 : *AM92 Select*  
over age 65 : *PFA92C20*

*Interest* : 4% *pa*

*Expenses* : *none*

(8 marks)

b) On 1 January 2002, a life insurance company issued whole life increasing assurances to life then aged 45 *exact*. The initial sum assured was £20,000, which increased by £2,000 on each policy anniversary. Benefit are payable at the end of year of death. Premiums are payable annually in advance for a maximum of 20 *years*, ceasing on earlier death.

On 1st January 2018, there were 378 policies in force and, during 2018, 4 of these policyholders died.

i) Calculate the mortality profit during 2018, assuming the insurance company uses the following basis for both premiums and reserves. (9 marks)

*Mortality* : AM92 Ultimate

*Interest* 4% per annum

*Expenses* none

ii) Explain why the result in part (i) has arisen (3 marks)

### 3 Question Three

a) A life office sells 5 – year term assurance policies to lives aged 60. Each policy has a sum assured of Kshs.150,000 payable at the end of year of death. Premiums of Kshs.30,000 are payable annually in advance throughout the 5 – year term or until earlier death.

Let  $L$  denote the present value of the insurer’s loss on one of these policies, at policy outset, ignoring expenses.

i) Write down an expression for  $L$ .

ii) Assuming AM92 Ultimate mortality and  $5\frac{1}{2}\%$  pa interest, calculate the expected value and standard deviation of  $L$ .

(12 marks)

b) i) Describe in words the difference between the functions  $(I\bar{A})_{x:\overline{n}|}$  and  $(\overline{IA})_{x:\overline{n}|}$

ii) Determine, showing all your working,  $(I\bar{A})_{x:\overline{15}|}$

Basis:

*Force of Mortality*  $\mu_x = 0.02$  for all  $x$

*Force of interest* 3%

(8 marks)

### 4 Question Four

a) A woman aged 67 exact takes out an annuity that makes monthly payments in arrears. The first monthly payment is £1,500, and payments increase by 0.23726% each month.

Calculate the expected present value of the annuity using the following basis:

*Mortality* PFA92C20  
*Interest* 7% per annum (4 marks)

b) A 10 – year term assurance with a sum assured of £500,000 payable at the end of year of death, is issued to a male aged 30 for a level annual premium of £330.05. Calculate the **prospective** and **retrospective** reserves at the end of the sixth policy year, *ie* just before the seventh premium has been paid, assuming *AM92 ultimate Mortality* and 4% *pa interest*. Ignore Expenses. (10 marks)

c) Describe the main features of unit-linked policies. (6 marks)

## 5 Question five

a) A life insurance company issues a three-year unit-linked endowment assurance policy to a life aged 58 exact under which level premium of £3,000 are payable annually in advance throughout the term of the policy or until earlier death. The premium allocation rate (%) at time  $t$  is given by:

$$75 + 20t$$

where  $t = 0, 1$  and  $2$ .

The units are subject to a bid-offer spread of 5%. An annual management charge of 0.75% of the bid value of the units is deducted at the end of each policy year. Management charges are deducted from the unit fund before any death, surrender or maturity benefits are paid.

If the policyholder dies during the term of the policy, the death benefit of £9,000 or the bid value of the units if higher, is payable at the end of the policy year of death. The policyholder may surrender the policy only at end of each policy year. On surrender at the end of the policy year or on survival to the end of the term, the current bid value of the units is payable.

The company uses the following assumptions in carrying out profit test of this contract:

The rate of growth of assets in the unit fund	4% <i>pa</i>
Rate of interest on non-unit fund cashflows	2% <i>pa</i>
Mortality	<i>AM92 Select</i>
Surrender	10% at end of the first, second and third policy year only
Initial expenses	£275
Renewal expenses	£70 per annum on the second and subsequent premium dates
Initial Commission	5% of first premium
Renewal Commission	2% of second and subsequent year's premiums
Risk discount rate	6% per annum

i) Calculate the profit margin for the policy on the assumption that the company does not zeroise future expected negative cashflows. (13 *marks*)

Suppose the company sets up reserves in order to zeroise future negative expected cashflows.

ii) Calculate the profit for the policy allowing for the cost of setting up these reserves. (4 *marks*)

b) Calculate  $A_{50:\overline{4}|}$

Basis:

*Mortality*  $q_{50} = 0.05$   
 $q_{51} = 0.06$

$q_{51+t} = 1.1q_{50+t}$  for  $t \geq 1$

*Interest* 6% *per annum*  
(3 *marks*)