KISII UNIVERSITY

UNIVERSITY EXAMINATIONS

MAIN/ELIMU CENTRE CAMPUSES

SECOND YEAR EXAMINATION FOR THE AWARD OF THE DEGREE OF BACHELOR OF EDUCATION [SCIENCE AND ARTS], BACHELOR OF SCIENCE [ACTUARIAL SCIENCE, ECONOMICS AND STATISTICS, ANALYTICAL CHEMISTRY, APPLIED STATISTICS AND COMPUTER SCIENCE (INTEGRATED)]

SECOND SEMESTER 2022/2023 [JAN -APRIL 2023]

MATH 241-PROBABILITY AND STATISTICS II

STREAMS: Y2 S2 KUCCPS/SSP **TIME:** $2HOURS$

INSTRUCTIONS

1. Do not write anything on this paper

2. Answer Question ONE and any other TWO questions.

3. Show ALL your workings and steps.

QUESTION ONE [COMPULORY (30marks)]

(a) If $P(E) = 0.9$ and $P(F) = 0.8$, show that $P(EF) \geq 0.7$. Hence show that in general $P(EF) \geq P(E) + P(F) - 1$. [**5 marks]**

(b) Suppose $Y_1, Y_2, ..., Y_n$ is an i.i.d sample from

$$f(y) = \begin{cases} p^y (1-p)^{1-y}, \ y = 0,1 \\ 0, \ elsewhere \end{cases}.$$

That is $Y_1, Y_2, \ldots, Y_n$ are Bernoulli (p) random variables. Find the distribution of $U = Y_1 + Y_2 \ldots + Y_n$ by the method of moment generating functions. **[6 marks]**

(c)) Suppose $X \sim U\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Show that the distribution of the random variable $Y = \tan X$ is given by $\frac{1}{\pi(1+y^2)}$ which is a standard Cauchy distribution . **[6 marks]**

(d) The probability function of a random variable X is given by

$f(x) = \begin{cases} 2xe^{-x^2}, & x > 0 \\ 0, & otherwise \end{cases}$. Find the probability density for the random variable $Y = X^2$. **[4 marks]**

(e) The joint probability function of two random variables X and Y is given by

$$f(x,y) = \begin{cases} \frac{1}{42}(2x+y), & x = 0,1,2;\ y = 0,1,2,3 \\ 0, & otherwise \end{cases}$$

Find the joint probability density function of $U = X - Y$ and $V = X + Y$

[6 marks]

(f) Suppose that X is a random variable whose probability distribution is given by

$$f(x) = \begin{cases} \frac{1}{4}, & x = 1,2,3,4. \\ 0, & otherwise \end{cases}$$

Find the probability distribution of $Y = 2X + 1$.

[3marks]

QUESTION TWO (20marks)

(a)Let Y denote the sum of the items of a random sample of size 100 from a Bernoulli distribution with parameter $p = \frac{1}{2}$. Find the approximate value of $P(Y = 48,49,50,51,52)$. **[5marks]**

(b) Let S^2 be the sample variance of a random sample of size 6 from the normal distribution $N(\mu,12)$. Find $P(2.30 < S^2 < 22.2)$. **[5marks]**

(c) Let $Y_1 < Y_2 < .Y_3 < Y_4$ denote the order statistics of a random sample of size n= 4 from a distribution having p.d.f $f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & elsewhere \end{cases}$.

Find:

(i) The p.d.f of the first order statistic. **[3marks]**

(ii) The p.d.f of the fourth order statistic. **[3marks]**

(iii) $P(0.5 < Y_3)$ **[4marks]**

QUESTION THREE (20marks)

(a) The Gamma function of α is defined as $\Gamma(\alpha) = \int_0^\infty t^{\alpha-1} e^{-t} dt$.

Using the transformation $Y = \beta X$, derive the gamma distribution with parameters α and β. Hence find $E(X)$ **[12 marks]**

(b) The Beta function is defined by $B(a,b) = \int_0^1 t^{a-1}(1-t)^{b-1} dt$. Use the transformation $T = \frac{1}{Y}$ to show that the beta distribution of the second kind is $B(a,b) = \int_0^\infty \frac{y^{a-1}}{(1+y)^{a+b}} dy$. Hence determine its mean and variance **[8marks]**

QUESTION FOUR (20 marks)

(a) Let X_1 and X_2 have the joint p.d.f:

$$f(x_1,x_2)=\begin{cases}\frac{x_1x_2}{36}, x_1=1,2,3; x_2=1,2,3\\ 0, otherwise\end{cases}$$. Find the marginal p.d.f of Y_1 given

that $Y_1 = X_1X_2$ and $Y_2 = X_2$. **[10 marks]**

(b) A random variable X has probability distribution given by

$$f(x)=\frac{x^{\alpha-1}e^{-\frac{x}{\beta}}}{\beta^{\alpha}\Gamma\alpha},\ x>0 \text{ and zero elsewhere.}$$

Use the method of moment generating function to derive the distribution of $Y=h(X)=\frac{2X}{\beta}$ **[10marks]**

QUESTION FIVE (20 marks)

(a) A random variable X has probability distribution given by

$$f(x)=\begin{cases}\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, & -\infty<x<\infty\\ 0, & \text{elsewhere}\end{cases}$$

Find the moment generating function of X. Hence show that the mean and variance of X are respectively μ and σ^2 **[13marks]**

(b) Suppose X has a binomial distribution given by

$$P(X=x)=\binom{n}{x}p^xq^{n-x}\ ,x=0,1,2,...,n$$

Show that the characteristic function of X is given by $\phi(t)=(q+pe^{it})^n$. **[7marks]**