KISII UNIVERSITY

UNIVERSITY EXAMINATIONS

SECOND YEAR EXAMINATION FOR THE AWARD OF THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS AND APPLIED STATISTICS

SECOND SEMESTER 2022/2023

[JANUARY-APRIL, 2023]

MATH 210: LINEAR ALGEBRA I

STREAM: Y2S2 **TIME: 2 HOURS**

DAY: WEDNESDAY, 3:00 – 5:00 PM **DATE: 29/03/2023**

INSTRUCTIONS

1. **Do not write anything on this question paper.**
2. ***Answer question ONE and any other TWO questions.***

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Calculate the determinant of the following matrices:

i) $A = \begin{pmatrix} \frac{1}{2} & \frac{2}{3} \\ \frac{1}{3} & -\frac{3}{5} \end{pmatrix}$ ii) $A = \begin{bmatrix} 1 & 5 & -2 \\ 3 & -1 & 4 \\ -3 & 6 & -7 \end{bmatrix}$ (7marks)

b) Find the eigen values of the matrix $A = \begin{pmatrix} 5 & 2 \\ 9 & 2 \end{pmatrix}$

(4marks)

c) Use Cramer's rule to solve:

i) $12u + 8a = 52$
$-16u + 6a = -36$ (3marks)

ii) $a + 2b - 3c = 3$
$2a - b - c = 11$
$3a + 2b + c = -5$ (6marks)

d) Find the vector equation of a line which passes through the points $A(4, 3, 1)$ and $B(2, 1, 1)$.

(5marks)

e) Find the distance between the vectors $\boldsymbol{u} = (1,2,3)$ and $\boldsymbol{v} = (1, 4, -1)$ (5marks)

QUESTION TWO (20MARKS)

a) Use Gauss-Jordan method to solve:

$$x + y + z = 4$$
$$2x - 3y + 4z = 33$$
$$3x - 2y - 2z = 2$$
(6marks)

b) Reduce the matrix to canonical form:

$$A = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$$ (7marks)

Find the area of the triangle having the vertices at the points $A(1,0)$, $B(2,2)$ and $C(4,3)$ (7marks)

QUESTION THREE (20MARKS)

a) Use the augmented matrix to find the inverse of the matrix

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$ (7marks)

b) Solve the equation $\begin{vmatrix} 4 & -x \\ 5 & 2x \end{vmatrix} = 4$ (3marks)

c) Find the adjoint of the matrix

$$A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$$ (7marks)

d) Determine the rank of the matrix $A = \begin{bmatrix} 1 & -1 & 3 \\ 2 & 0 & 4 \\ -1 & -3 & 1 \end{bmatrix}$ (3marks)

QUESTION FOUR (20MARKS)

a) Find the co-factors of the matrix A, if:

$$A = \begin{bmatrix} -2 & 5 & 4 \\ 5 & 7 & 5 \\ 4 & 5 & -2 \end{bmatrix}$$ (8marks)

b) Find the value of m for $\begin{vmatrix} m+1 & 0 & 0 \\ 4 & m & 3 \\ 2 & 8 & m+5 \end{vmatrix} = 0$ (7marks)

c) Show that the vectors $\boldsymbol{u} = 3i - 4j$ and $\boldsymbol{v} = 4i - 3j$ are orthogonal. (5marks)

QUESTION FIVE (20MARKS)

a) Define a linear transformation and hence show that T: $R^3 \to R^2$ defined by:

$$T\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} X_1 + X_2 \\ X_2 - X_3 \end{bmatrix}$$ is a linear transformation. (8marks)

b) i) Define the term a symmetric matrix.. (2marks)

ii) Prove that a symmetric matrix of order 2 is diagonalizable. (5marks)

c) Find the point of intersection of the lines whose parametric equations are:

$$l_1: x = 3 - 2t, \qquad y = 4 + t$$

$l_1: x = 1 + 4t,\ y = 5 + 2t$ (5marks)