KISII UNIVERSITY

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

BACHELOR OF ACTUARIAL SCIENCE/APPLIED STATISTIS/ PURE MATHEMATICS/ APPLIED MATHEMATICS

COURSE CODE: BBAM 272/MATH 348/STAT 348

COURSE TITLE: OPERATIONS RESERCH I

FINAL EXAM APRIL 2023

***INSTRUCTIONS*: Answer question one and any other 2 questions in section**

SECTION A (30 MARKS)

QUESTION ONE (30 MARKS)

a) A transport company has two types of trucks, Type A and Type B. Type A has a refrigerated capacity of $20m^3$ and a non-refrigerated capacity of $40m^3$ while Type B has the same overall volume with equal sections for refrigerated and non-refrigerated stock. A grocer needs to hire trucks for the transport of $3000m^3$ of refrigerated stock and $4,000m^3$ of non-refrigerated stock. The cost per kilometer of a Type A is 30 and 40 for Type B. How many trucks of each type should the grocer rent to achieve the minimum total cost using graphical method? (6mks)

b) A plant manager has four subordinates, and four tasks to be performed. The subordinates differ in efficiency and the tasks differ in their intrinsic difficulty. This estimate of the times each man would take to perform each task is given in the effectiveness matrix below.

	I	**II**	**III**	**IV**
A	8	26	17	11
B	13	28	4	26
C	38	19	18	15
D	19	26	24	10

How should the tasks be allocated, so as to minimize the total man hours? (4mks)

c) Solve the following LPP using BIG M method (6mks)

$$minimize\ Z = 4x_1 + x_2$$
$$subject\ to: 3x_1 + x_2 = 3$$
$$4x_1 + 3x_2 \geq 6$$
$$x_1 + 2x_2 \leq 4$$
$$x_1, x_2 \geq 0$$

d) Explain special cases of simplex algorithm (4mks)

e) A project schedule has the following characteristics

Activity	1-2	1-3	2-4	3-4	3-5	4-9	5-6	5-7	6-8	7-8	8-10	9-10
Time	4	1	1	1	6	5	4	8	1	2	5	7

Construct the network model, perform forward and backward pass, critical path and project completion time (7mks)

f) Define linear independent and linear dependence (3mks)

SECTION B

QUESTION TWO (20 MARKS)

a) Write the dual of the following primal LPP question (4mks)

$$maximize\ Z = 5x_1 + 12x_2 + 4x_3$$
$$subject\ to:$$
$$x_1 + 2x_2 + x_3 \leq 10$$
$$2x_1 - x_2 + 3x_3 = 8$$
$$x_1, x_2, x_3 \geq 0$$

b) Solve the following question using simplex method (6mks)

$$maximize\ Z = 3x_1 + 2x_2$$

$$subject\ to:$$

$$-x_1 + 2x_2 \leq 4$$

$$3x_1 + 2x_2 \leq 14$$

$$x_1 - x_2 \leq 3$$

$$x_1, x_2 \geq 0$$

c) Construct the network model, perform forward and backward pass, critical path and project completion time (7mks)

Activity	Predecessor	Optimistic duration	Pessimistic duration	Most likely duration
O	-	0	0	0
A	O	12	18	15
B	O	6	12	9
C	A	9	15	12
D	B	6	18	9
E	B	18	36	30
F	A	9	15	12
G	C	36	42	36
H	D	42	54	48
I	A	6	18	12
J	H,G,E	3	9	6
K	F,J,I	3	9	6

d) Explain phases of project management (3mks)

QUESTION THREE (20 MARKS)

a) Solve the following LPP using Vogel's method and test for optimality (7mks)

Factory	Warehouse				Capacity
	W1	W2	W3	W4	
F1	21	16	25	13	11
F2	17	18	14	23	13
F3	32	27	18	41	19
Requirement	6	10	12	15	43

b) Write the dual of the following primal program (3mks)

$$minimize\ Z = 15x_1 + 12x_2$$

$$subject\ to:$$

$$x_1 + 2x_2 \geq 3$$

$$2x_1 - 4x_2 \leq 5$$

$$x_1, x_2 \geq 0$$

c) Solve the following question using BIG M method (5mks)

$$maximize\ Z = -2x_1 - x_2$$

$$subject\ to:$$

$$3x_1 + x_2 = 3$$

$$4x_1 + 3x_2 \geq 6$$

$$x_1 + 2x_2 \leq 4$$

$$x_1, x_2 \geq 0$$

d) A manufacturer of cylindrical containers receives tin sheets in widths of 30 cm and 60 cm respectively. For these containers the sheets are to be cut to three different widths of 15 cm, 21 cm and 27 cm respectively. The number of containers to be manufactured from these three widths are 400, 200 and 300 respectively. The bottom plates and top covers of the containers are purchased directly from the market. There is no limit on the lengths of standard tin sheets. Formulate the LPP for the production schedule that minimises the trim losses. (5mks)

QUESTION FOUR (20 MARKS)

a) A dairy plant has five milk tankers I, II, III, IV & V. These milk tankers are to be used on five delivery routes A, B, C, D, and E. The distances (in kms) between dairy plant and the delivery routes are given in the following distance matrix

	I	II	III	IV	V
A	160	130	175	190	200
B	135	120	130	160	175
C	140	110	155	170	185
D	50	50	80	80	110
E	55	35	70	80	105

How the milk tankers should be assigned to the chilling centers so as to minimize the distance travelled? (5mks)

b) A project schedule has the following characteristics

Job	1-2	1-3	2-4	3-4	3-5	4-5	4-6	5-6
Duration	6	5	10	4	4	6	2	9

Construct the network model, perform forward and backward pass, critical path and project completion time (5mks)

c) Use BIG M method to solve the following LPP program (7mks)

$$minimize\ Z = 4x_1 + 3x_2$$
$$subject\ to:$$
$$2x_1 + x_2 \geq 10$$
$$-3x_1 + 2x_2 \leq 6$$
$$x_1 + x_2 \geq 6$$
$$x_1, x_2 \geq 0$$

d) Highlight on characteristics of a linear program (3mks)

QUESTION FIVE (20 MARKS)

a) A farmer has 200 acres of land. He produces three products X, Y & Z. Average yield per acre for X, Y & Z is 4000, 6000 and 2000 kg. Selling price of X, Y & Z is Rs. 2, 1.5 & 4 per kg respectively. Each product needs fertilizers. Cost of fertilizer is Rs. 1 per kg. Per acre need for fertilizer for X, Y & Z is 200, 200 & 100 kg respectively. Labour requirements for X, Y & Z is 10, 12 & 10 man hours per acre. Cost of labour is Rs. 40 per man hour. Maximum availability of labour is 20,000 man hours. Formulate as LPP to maximise profit. (5mks)

b) Solve the following LPP using least cost model and test for optimality (7mks)

		Warehouse				Capacity
	W1	W2	W3	W4	W5	
P1	20	28	32	55	70	50
P2	48	36	40	44	25	100
P3	35	55	22	45	48	150
Demand	100	70	50	40	40	300

c) Define convex combination (3mks)

d) At the head office of there are five registration counters. Five persons are available for service.

	I	II	III	IV	V
1	30	37	40	28	40
2	40	24	27	21	36
3	40	32	33	30	35
4	25	38	40	36	36
5	29	62	41	34	39

How should the counters be assigned to persons so as to maximize the profit? (5mks)